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# Fundamental Results of Two-dimensional Generalized Canonical SC-Transform

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**Abstract:** This paper is concerned with the definition of two-dimensional (2-D) generalized canonical SC- transform it is extended to the distribution of compact support by using kernel method. We have discussed inversion theorem for that transform. Lastly we have proved Uniqueness theorem for that transform.

**Keywords:** 2-D canonical transform, 2-D sine-cosine transform, 2-D sine-sine transform, 2-D cosine-cosine transform, 2-D fractional Fourier transform, generalized function.

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## 1) Introduction : –

Now a days fractional Fourier transforms plays important role in information processing [5]. The fractional Fourier transform as an extension of the Fourier transform. It has been used many applications such as optical system analysis, filter design, solving differential equations. Phase retrieval and pattern recognition etc. [8] [3],In fact the fractional Fourier transform is special case of the canonical transform. The canonical transform is defined as

$$\{CT\,f(t)\}(s)=\frac{1}{\sqrt{2\pi ib}}\int_{-\infty}^{\infty}e^{-i\left(\frac{s}{b}\right)t}\,e^{\frac{i}{2}\left(\frac{a}{b}\right)t}\,f(t)\,dt \qquad b\neq 0 \qquad \ldots\ldots\ldots(1)$$

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$$= \sqrt{d} e^{\frac{i}{2}\left(cds^2\right)} f\left(d.s\right) \qquad b = 0$$

And the constraint that ad-bc=1 must be satisfied. The canonical transform defined above in (1) are all one-dimensional [1-D], in [1] [2], [10],[11],[12],[13],[14],[15], they have generalized them from one–dimensional into the (2-D) cases, [4] ,[06],[07].The two-dimensional canonical sine-cosine transform it is extended to the distribution of compact support by using kernel method [09] .

The two-dimensional canonical sine-cosine transform is defined as.

$$\{2DCSCT\ f(t,x)\}(s,w) = -i\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} . e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2} f(t,x)\, dxdt$$

When $b \neq 0$

Notation and terminology of this paper is as per [17], [18]. The paper is organized as follows. Section. 2 gives the definition of 2-D canonical sine-cosine transform on the space of generalized function in section. 3 inversion theorem is proved in section. 4 Uniqueness theorems proved lastly the conclusion is stated.

**2. Definition two Dimensional (2D) Generalized canonical sine-cosine transform** $[2DCSCT]$ **:**

Let $E^l\left(R\times R\right)$ denote the dual of $E\left(R\times R\right)$ therefore the generalized canonical sine-cosine transform of $f\left(t,x\right)\in E^l\left(R\times R\right)$ is defined as

$$\left\{2DCSCT\ f\left(t,x\right)\right\}\left(s,w\right) = \left\langle f\left(t,x\right), K_s\left(t,s\right)K_c\left(x,w\right)\right\rangle$$



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$$\{2DCSCT\ f(t,x)\}(s,w)$$
$$=(-i)\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

Where $K_s(t,s)=(-i)\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\sin\left(\frac{s}{b}t\right)$ when $b\neq 0$

$=\sqrt{d}\,e^{\frac{i}{2}(cds^2)}\delta(t-ds)$ when b = 0

and $K_c(x,w)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}\cos\left(\frac{w}{b}x\right)$ when $b\neq 0$

$=\sqrt{d}\,e^{\frac{i}{2}(cdw^2)}\delta(x-dw)$ when b = 0

where $\gamma_{E,k}\{K_s(t,s)K_c(x,w)\}=\sup_{\substack{-\infty<t<\infty\\ -\infty<x<\infty}}\left|D_t^k D_x^l K_s(t,s)K_c(x,w)\right|<\infty$

**3. Theorem :( Inversion)** If $\{2DCSCT\ f(t,x)\}(s,w)$ is canonical sine-cosine transform of $f(t,x)$ then

$$f(t,x)=-ie^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2}\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}.$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)\{2DCSCT\ f(t,x)\}(s,w)\,dsdw,$$

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**Proof:** The two dimensional canonical sine- cosine transform if $f(t,x)$ is given by

$$\{2DCSCT\ f(t,x)\}(s,w)$$
$$=-i\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$f(s,w)=\{2DCSCT\ f(t,x)\}(s,w)$$

$$\therefore f(s,w)$$
$$=-i\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$f(s,w).\sqrt{2\pi ib}\sqrt{2\pi ib}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}.e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}$$
$$=-i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$\therefore C_1(s,w)=f(s,w)\sqrt{2\pi ib}\sqrt{2\pi ib}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}$$

And $g(t,x)=e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)$

$$C_1(s,w)=-i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(t,x).\sin\left(\frac{s}{b}t\right).\cos\left(\frac{w}{b}x\right)dxdt$$

$$C_1(s,w)=\{2DCSCTg(t,x)\}\left(\frac{s}{b},\frac{w}{b}\right)$$


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Where $\left\{2DCSCTg\left(t,x\right)\right\}\left(\frac{s}{b},\frac{w}{b}\right)$ is 2D Canonical sine-cosine transform of *g(t,x)*.2D canonical sine -cosine transform *g(t,x)* with argument

$\therefore \frac{s}{b}=\eta$ and $\frac{w}{b}=\xi$ Therefore, $\frac{ds}{b}=d\eta$ and $\frac{dw}{b}=d\xi$

$\therefore C_1(s,w)=\left\{2DCSCTg\left(t,x\right)\right\}\left(\eta,\xi\right)$

By using inversion formula we get $\therefore g(t,x)=-i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_1(s,w)\sin(\eta t)\cos(\xi x)d\eta d\xi$

$$g(t,x)=-i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(s,w)\sqrt{2\pi ib}\sqrt{2\pi ib}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\sin(\eta t)\cos(\xi x)d\eta d\xi$$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)=-i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(s,w)\sqrt{2\pi ib}\sqrt{2\pi ib}\ e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)\frac{ds}{b}\frac{dw}{b}$$

$$e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)$$
$$=-i\sqrt{2\pi ib}\sqrt{2\pi ib}\frac{1}{b}\frac{1}{b}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)f(s,w)\,dsdw$$

$$f(t,x)$$
$$=-ie^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-\frac{i}{2}\left(\frac{a}{b}\right)x^2}\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)w^2}\sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right)\left\{2DCSCT\ f(t,x)\right\}(s,w)\,dsdw$$


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**3. Theorem:(Uniqueness)** If $\{2DCSCT\ f(t,x)\}(s,w)$ and $\{2DCSCT\ g(t,x)\}(s,w)$ are 2D canonical sine-cosine transform and

$\sup pf \subset s_a$, and $s_b$ and, $\sup p\,g \subset s_a, and\ s_b$

Where $s_a = \{t : t \in R^n,\ |t| \le a,\ a > 0\}$ and $s_b = \{x : x \in R^n,\ |x| \le b, b > 0\}$

If $\{2DCSCT\ f(t,x)\}(s,w) = \{2DCSCT\ g(t,x)\}(s,w)$

then, $f = g$ in the sense of equality in $D'(I)$

**Proof:** By inversion theorem $f - g$

$$= \left( -ie^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2} \sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2} \sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right) \{2DCSCT\ f(t,x)\}(s,w)\,dsdw \right)$$

$$- \left( -ie^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2} \sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2} \sin\left(\frac{s}{b}t\right)\cos\left(\frac{w}{b}x\right) \{2DCSCT\ g(t,x)\}(s,w)\,dsdw \right)$$

$$\therefore\ f - g = -i\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\, e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{-\frac{i}{2}\left(\frac{a}{b}\right)x^2} \int_{-\infty}^{\infty} e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2} \sin\left(\frac{s}{b}x\right)\cos\left(\frac{w}{b}x\right) \left[\{2DCSCT f(t,x)\} - \{2DCSCT g(t,x)\}\right] dsdw$$

Thus $f = g$ in $D'(I)$


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**Conclusion: -** In this paper two-dimensional canonical sine-cosine is Generalized in the form the distributional sense, we have inversion theorem for this transform is proved. Lastly uniqueness theorem is also proved.

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